Written Exam for the M.Sc. in Economics Autumn 2013 (Fall Term)

Financial Econometrics A: Volatility Modelling

Final Exam: Masters course

Exam date: **20/2-2014**

3-hour open book exam.

Please note there are a total of **8** questions which should **all be replied to**. That is, **4** questions under *Question A*, and **4** under *Question B*.

Total numbers of pages (including this one): 4

Please also note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by "eksamen på dansk" in brackets, you must write your exam paper in Danish. If you are in doubt about which title you registered for, please see the print of your exam registration from the students' self-service system.

Question A:

Question A.1: A novel class of GARCH (nGARCH) models may be presented as follows:

Consider y_t returns at time t, and σ_t^2 the conditional volatility, where σ_t^2 is specified conditional on the past variables of y_t and σ_t^2, that is $\mathcal{F}_{t-1} = \left(y_{t-1}, y_{t-2}, ..., y_0; \sigma_{t-1}^2 ..., \sigma_1^2\right)$. We specify first the equation for the (log) density $\log p(\cdot)$ of y_t conditional on σ_t^2 and $\mathcal{F}_{t-1}$. In the Gaussian case, we have the usual.

$$\log p\left(y_t|\sigma_t^2, \mathcal{F}_{t-1}\right) = -\tfrac{1}{2}\log\sigma_t^2 - \tfrac{1}{2}y_t^2/\sigma_t^2.$$

What differs in the nGARCH model is the specification of σ_t^2, where in general this is specified as,

$$\begin{aligned}
&\sigma_t^2 = \gamma_1 + \gamma_2 s_{t-1} + \gamma_3\sigma_{t-1}^2, \\
&s_t = \delta_t\left[E\left(\delta_t^2|\mathcal{F}_{t-1}\right)\right]^{-1}, \text{ and } \delta_t = \partial\log p(\cdot)/\partial\sigma_t^2.
\end{aligned}$$

Show that the nGARCH as specified above actually can be rewritten as a classic GARCH model, that is as,

$$\sigma_t^2 = \gamma_1 + \gamma_2 y_{t-1}^2 + (\gamma_3 - \gamma_2)\,\sigma_{t-1}^2$$

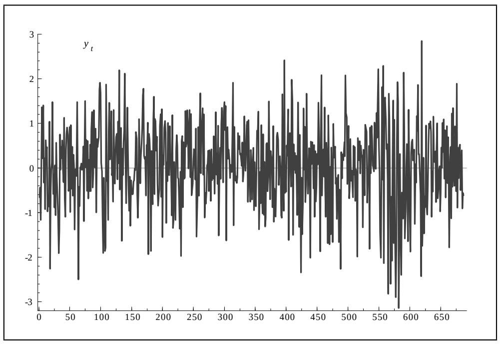
Question A.2: Establish that if $0 \leq \gamma_2 \leq \gamma_3 < 1$ then σ_t^2 is weakly mixing and has a stationary solution with $E\sigma_t^2 < \infty$.

Question A.3: It follows that with S_{γ_2} the score in the γ_2 direction (up to a constant), then

$$S_{\gamma_2} = \sum_{t=1}^{T}\left(\tfrac{y_t^2}{\sigma_t^4} - \tfrac{1}{\sigma_t^2}\right)\partial\sigma_t^2/\partial\gamma_2.$$

Find $\partial\sigma_t^2/\partial\gamma_2$ and show that if $\gamma_3 = \gamma_2$, then $\frac{1}{\sqrt{T}}S_{\gamma_2}$ is asymptotically Gaussian distributed if $E\sigma_{t-1}^4 < \infty$. Explain what this can be used for.

Question A.4: The figure below shows returns y_t for $t = 0, 1, ..., 687$, and Table 1 output from estimation of the GASGARCH. Comment on all output in the tab



Return series y_t

nGARCH	$\hat{\gamma}_2 = 0.1$	$\hat{\gamma}_3 = 0.98$	LM-ARCH p-value: 0.76

Table 1: Estimation by nGARH

Question B:

Consider the threshold SV (TSV) model as given by,

$$y_t = \sigma_t z_t \tag{B.1}$$

$$\log \sigma_t^2 = \rho \delta_t \log \sigma_{t-1}^2 + \xi_t, \tag{B.2}$$

where z_t and ξ_t are independent, with z_t i.i.d.N$(0, 1)$ and ξ_t i.i.dN$(0, \sigma_\xi^2)$. Moreover, $\delta_t = 1(\log \sigma_{t-1}^2 \leq \gamma)$, for some positive threshold parameter $\gamma > 0$. That is $\delta_t = 1$ if $\log \sigma_{t-1}^2 \leq \gamma$ and zero otherwise.

Question B.1: Show that $\log \sigma_t^2$ is weakly mixing and has a stationary representation for $|\rho| < 1$, and discuss what this implies, if anything, for the joint evolution of y_t and $\log \sigma_t^2$?

Question B.2: Set $Y_t := \log |y_t| - \mu$, where $\mu = E \log |z_t|$. Moreover, set $X_t := \log \sigma_t$. It follows that the system for Y_t and X_t can be written as,

$$Y_t = X_t + \varepsilon_t, \quad X_t = \rho \left(X_{t-1} \right) X_{t-1} + \xi_t.$$

Here ε_t is i.i.d.$(0, \sigma_\varepsilon^2)$. Moreover, $\rho(X_{t-1}) = \rho 1(X_{t-1} \leq \gamma/2)$.

Argue that ε_t is not Gaussian, but that ε_t is an iid sequence with mean zero and find the variance.

Explain why one would be interested in computing $X_{t|t-1} = E(X_t | Y_{1:t-1})$ as a function of $X_{t-1|t-1} = E(X_{t-1} | Y_{1:t-1})$.

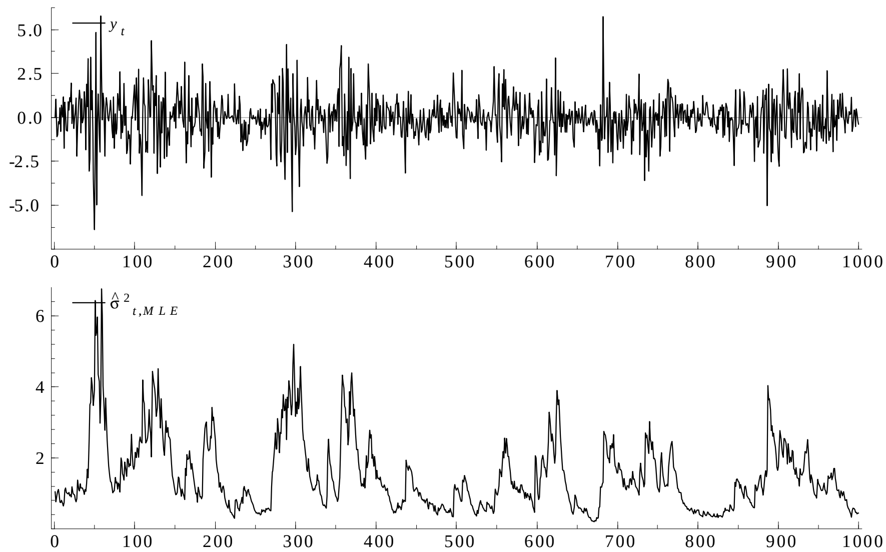
Explain also why the *linear* Kalman filter would not work.

Question B.3: Provide an outline of how you would find the MLE of $\theta = (\rho, \gamma, \sigma_\xi^2)$ based on the particle filter.

State the missing "....." part in the piece of code below from ox for the Bootstrap particle filter.

```
DrawProposal(x_1)
{
decl x;
x = ......+ sqrt(sigma)*rann(1,1);
return x;
}
```

Question B.4: The figure below shows a return series y_t, $t = 1, 2, ..., T$ together with $\hat{\sigma}_{t,MLE}^2$ based om MLE using the particle filter. Explain how $\hat{\sigma}_{t,MLE}^2$ could be obtained in different ways by using a particle filter. Elaborate.



Return series y_t and estimated σ_t^2